

Department of Geodetic Science and Surveying

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Improvement of the Earth's Gravity Field from Terrestrial and Satellite Data

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1. Introduction

The work being performed under this grant is directed towards the improvement of our knowledge of the gravity field and in the analysis of quantities that depend on the gravity field.

2. Progress in our Research Effort

In the following paragraphs we outline the work that has been carried out in this reporting period.

2.1 Terrestrial Gravity Data

We have now completed the update of our $1^\circ \times 1^\circ$ terrestrial gravity data base. This data set, the June 1986 set, contains 48955 mean anomalies and their accuracy. This field contains 4479 anomalies that were not in our previous (January 1983) data set. The location of all anomalies is shown in Figure 1. In developing this field special care was taken to place realistic accuracy estimates on the anomalies where possible. Comparisons of terrestrial estimates with newly derived estimates from altimeter data enabled us to select the most reliable values in ocean regions.

In this anomaly set 5684 anomalies have been estimated through geophysical correlation techniques. In the total set of anomalies, 19584 anomalies did not come from the Defense Mapping Agency Aerospace Center.

For the first time in our studies, we have started a $30' \times 30'$ mean anomaly data base. This data set will be used for the development of high degree spherical harmonic expansions and for precise geoid computations. The first version of this data set contains 31787 anomalies. The areas in which these anomalies are located is shown in Figure 2.

A report describing the updating of the $1^\circ \times 1^\circ$ field and the creation of the $30' \times 30'$ file has been prepared by Despotakis. The specific reference can be found in Section 5.

2.2 GRM Studies

Our studies related to the GRM mission have primarily considered the local recovery of gravity anomalies on the surface of the earth based on satellite to satellite tracking or gradiometer data. Two studies were completed in this reporting period. The first study was prepared by Wichiencharoen (1985). The abstract of his report is as follows:

A simulation study was used to estimate the accuracy of 1° -mean anomalies which could be recovered from the Geopotential Research Mission (GRM) data. The earth's gravity field was defined by a set of potential coefficients of Rapp (1981) to degree 180. Line of sight accelerations were used as measurements in the recovery. The line of sight acceleration can be written in terms of potential coefficients, which makes data generation at a regular grid interval

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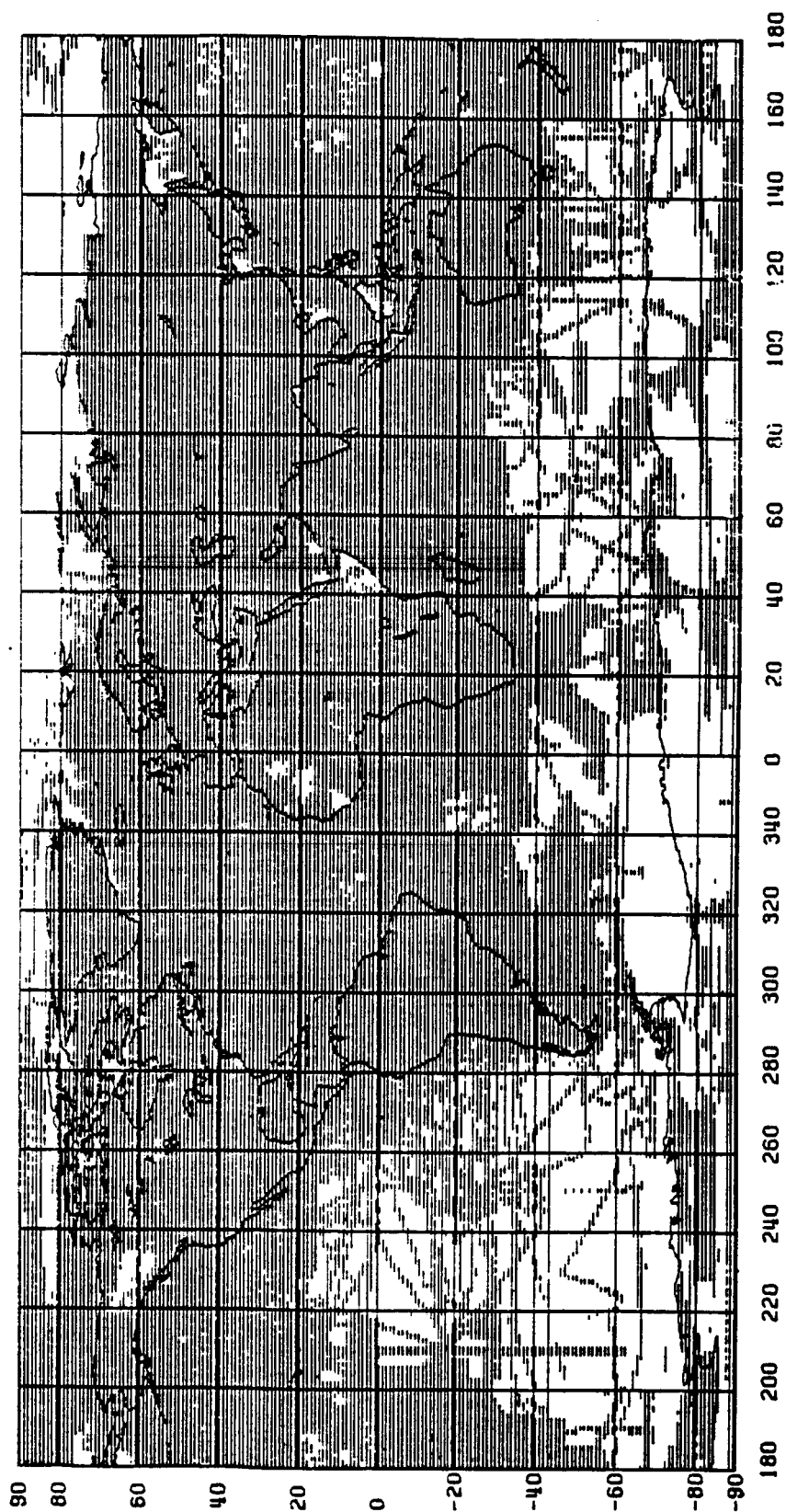


Figure 1. Location of the 48955 1°x1° Anomalies in the June 1986 Field

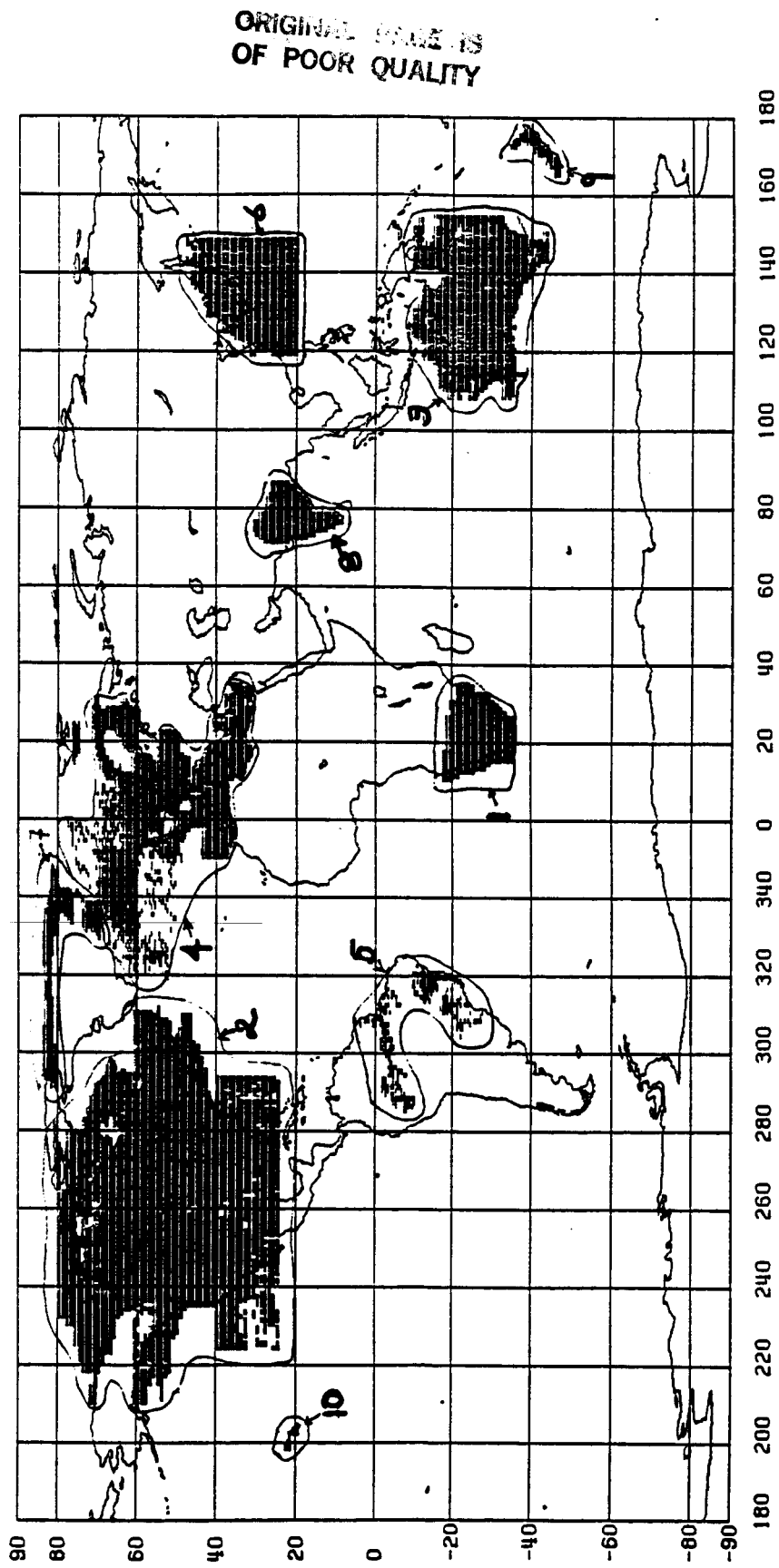


Figure 2. Location of the 31787 Anomalies of the Original 30'x30' Data Set

very efficient. An altitude of 160 kilometers above a spherical earth and a separation of 200 kilometers were used in the data simulation. Three methods were used in the recovery: the least squares collocation method; a numerical integration procedure proposed by Rummel (1982) or the Rummel procedure; and the least squares adjustment. In the least squares adjustment, a Fourier series of fictitious surface densities was used to represent a local gravity field; the Fourier coefficients were the unknowns. The Fourier coefficients were then used to compute a gravity anomaly in a very simple way. Three techniques of least squares adjustment were employed: the method of observation equations; the method of observation equations with weighted parameters; and the singular value decomposition method. After 1°-mean anomalies in a selected area were recovered by these estimation methods, they were compared with the true values (directly computed from the 180-field) to obtain the accuracy estimate. The result of the Rummel procedure was unacceptable because the fundamental assumption of the procedure, an isotropic functional relationship between the measurement and the unknown, was not fulfilled in the low-low satellite to satellite tracking squares adjustment indicated that an accuracy of 2.5 milligals for 1°-mean anomalies was possible in an area of the smooth gravity field. The accuracy of the recovery over the rough gravity field was about 30 milligals.

The second study was carried out by Robbins (1985). The abstract of his report is as follows:

An autonomous spaceborne gravity gradiometer mission is being considered as a post Geopotential Research Mission project. The introduction of satellite gradiometry data to geodesy is expected to improve our solid earth gravity models. This study explores the possibility of utilizing gradiometer data for the determination of pertinent gravimetric quantities on a local basis. The analytical technique of least squares collocation is investigated for its usefulness in local solutions of this type. It is assumed, in the error analysis, that the vertical gravity gradient component of the gradient tensor is used as the raw data signal from which the corresponding reference gradients are removed to create the centered observations required in the collocation solution. The reference gradients are computed from a high degree and order geopotential model. The solution can be made in terms of mean or point gravity anomalies, height anomalies, or other useful gravimetric quantities depending on the choice of covariance types. Selected for this study were 30'x30' mean gravity and height anomalies. Existing software and new software are utilized to implement the collocation technique. It was determined that satellite gradiometry data at an altitude of 200 km can be used successfully for the determination of 30'x30' mean gravity anomalies to an accuracy of 9.2 mgal from this algorithm. It is shown that the resulting accuracy estimates are sensitive to gravity model coefficient uncertainties, data reduction assumptions and satellite mission parameters.

2.3 Geoid Studies

During past research studies we have attempted to improve the way in which geoid undulations have been computed. In the past few years this study has become more feasible due to the use of high degree reference fields, and refined computational procedures. In this reporting period

Despotakis has developed numerous procedures and data with the intent of carrying out computations at the laser stations in the SL6 system. Comparisons of the computed undulations with the values found from the given ellipsoidal heights and the orthometric heights can shed light on the most accurate method to use for the computations. Despotakis has prepared the following report on his research:

"New methods for precise undulation computations have been tested with the ultimate goal being to use these methods for the precise computation of the geoid undulations at some Laser stations distributed around the world. All the methods combine terrestrial gravity anomalies within a limited cap of radius r_c surrounding the computation point, with a high degree geopotential model, basically through the integral formulas of Stokes' equation.

LASER STATIONS:

The total number of Laser stations that was selected is 59. Geoid undulations will be computed for those stations for which sufficient local terrestrial gravity information exists. The coordinates of the stations have been transformed to the SL6 system. One more transformation was done from the SL6 to the GRS'80 reference ellipsoid, since the computed geoid undulation will finally refer to the GRS'80 reference ellipsoid.

GRAVITY DATA:

The following gravity data are available to us under the form of Magnetic Tapes:

- United States: Tape GS202 with point gravity data; Tape GS308 with detailed 30"x30" elevation information
- Europe: Tape EUROP with 6'x10' mean free-air anomalies
- Australia: Tape GS324 with point gravity data
- Japan: Tape GS246 With 10'x10' mean free-air anomalies.

UNDULATION COMPUTATION:

Two new methods were implemented for the precise undulation computations:

- a) Molodenskii's modified method
- b) Sjöberg's least squares method.

Together with the new methods, the traditional method of Stokes' and Meissl's modification have been tested. The (total) global RMS undulation error has been computed for all the methods considering the following error sources:

<u>ERROR</u>		<u>REASON</u>
commission	c	erroneous potential coefficients
ommission	o	limited degree of expansion of geopotential model
discretion	d	the numerical integration is done with mean values
propagation	p	erroneous gravity data

The total error is $\delta\bar{N} = (c^2 + o^2 + d^2 + p^2)^{1/2}$, assuming no correlation between the various error sources.

For the numerical computations the potential coefficients of expansion (OSU86F) together with their standard deviations were used, up to degree $M=180$. In Figure 3 we see the total error of the various methods. If M is the degree of expansion and \bar{n} is the number of harmonics removed from the

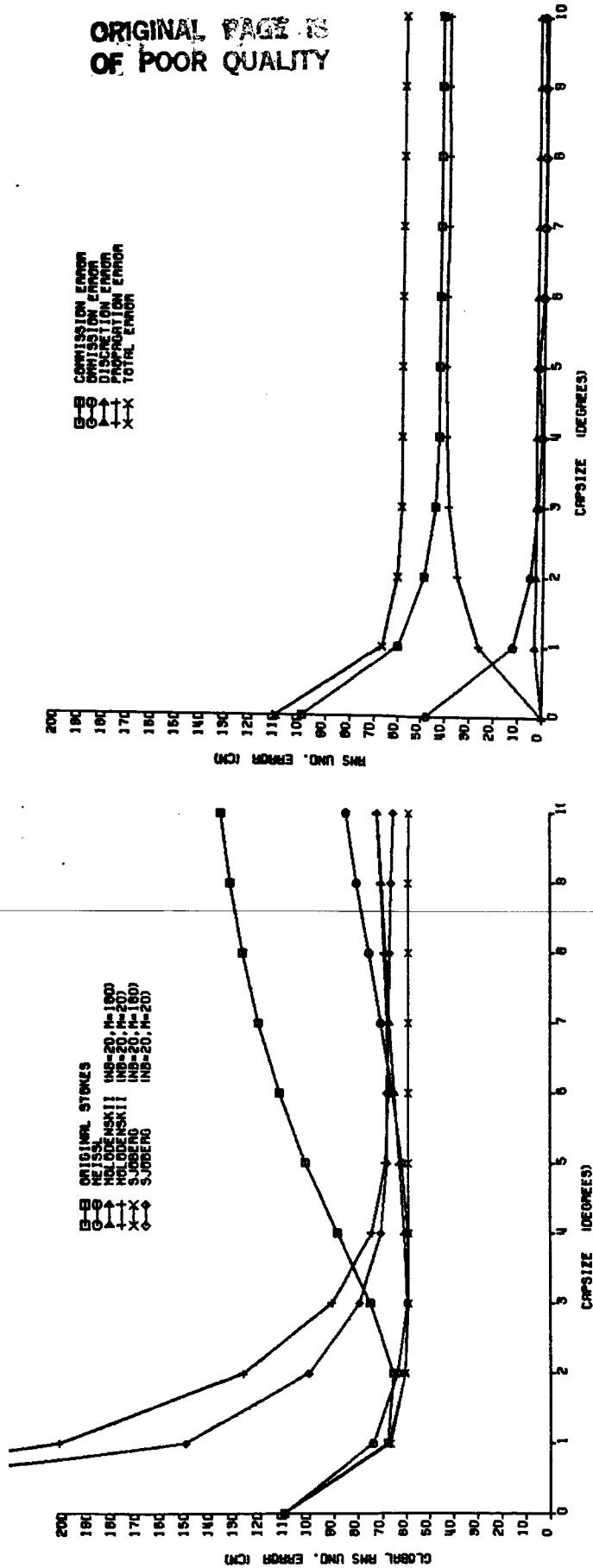


Figure 3. Total RMS undulation error for the known methods

Figure 4. Error analysis for Sjöberg's method ($\bar{n}=20$, $M=180$)

Stokes' function, we have (in correspondence with Figure 3):

<u>METHOD</u>	<u>\bar{n}</u>	<u>M</u>
Original Stokes	0	180
Meissl	0	180
Molodenskii	20	180
Molodenskii	20	20
Sjöberg	20	180
Sjöberg	20	20

In Figure 4 we can see the various error contributions to the total error for Sjoberg's method ($\bar{n}=20$, $M=180$). The major contribution comes from the erroneous gravity data within the cap.

Recent studies showed that Sjoberg's method achieves a precision of ≈ 58 cm at the capsize of 2° and retain approximately the same precision even if the cap is extended to $\psi_c=100^\circ$! Also, approximately the same precision is obtained even if $\bar{n}=5$ which means we obtain the same accuracy with less computational effort.

Also, ellipsoidal corrections have been derived for the new methods and they will be computed up to $M=36$.

Finally, the atmospheric correction has been taken into account for the implementation of the new methods, though the use of a second order polynomial of the elevation of the gravity stations."

2.4 Potential Coefficient Determinations

We have created the first spherical harmonic expansions of the gravity field to degree 360. This has been done on the basis of a) a rigorous adjustment of terrestrial gravity data in $1^\circ \times 1^\circ$ cells, altimeter derived anomalies in $1^\circ \times 1^\circ$ cells and the GEML2' potential. This solution is described in a report by Rapp and Cruz (1986a). This specific research was not supported by this NASA grant; b) the $30' \times 30'$ mean anomalies from terrestrial data and as derived from altimeter data. In fact we have computed two high degree fields that are designated OSU86E and OSU86F. The OSU86E field excludes most geophysically predicted anomalies while the OSU86F field includes such anomalies. A report describing these new solutions is now being prepared. A brief description of the new solutions was made at the Fall 1986 AGU meeting (Rapp and Cruzz, 1986b).

The techniques for carrying out the combination of satellite and terrestrial data, and the estimation of high degree reference fields is described in Rapp (1986a). A general view of the gravity estimation is given in Rapp (1986b).

2.5 Other Studies

During this time studies have gone on related to the representation of the earth's gravity field using ellipsoidal harmonics. Progress has been slow

in this development.

3. Future Progress

During the next reporting period we expect to have ready the reports on undulation computations at laser stations, and the report on the spherical harmonic expansions to degree 360. If sufficient progress is made we may have a report on the ellipsoidal harmonic problem.

4. Personnel

During this reporting period the following persons have participated in the grant research:

Richard H. Rapp, Principal Investigation
C. Wichiencharoen, Graduate Research Associate
J. Robbins, Graduate Research Associate
V. Despotakis, Graduate Research Associate
T. Engelis, Graduate Research Associate
S. Pertsinidu, Graduate Research Associate
J. Cruz, Post-Doctoral Researcher.

5. Reports and Papers

Despotakis, V., The Development of the June 1986 1°x1° and the August 1986 30'x30' Terrestrial Mean Free Air Anomaly Data Bases, Internal Report, Dept. of Geodetic Science and Surveying, The Ohio State University, 1986.

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Robbins, J., Least Squares Collocation Applied to Local Gravimetric Solutions from Satellite Gradiometer Data, Report No. 368, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, 100p, 1985.

Wichiencharoen, C., Recovery of 1° Mean Anomalies in a Local Region from a Low-Low Satellite to Satellite Tracking Mission, Report No. 363, Dept. of Geodetic Science and Surveying, The Ohio State University, 128p, 1985.